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**Considerations and Experiences in  
Developing a Finite Element Buttock Model  
for Seating Comfort Analysis**

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**Air Force Research Laboratory  
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# Considerations and Experiences in Developing a Finite Element Buttock Model for Seating Comfort Analysis

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## ABSTRACT

The comfort of seat cushions has become important in many of today's high-performance USAF fighter and tactical aircraft. Experimental investigations have found that there exists a strong relationship between the human subjective discomfort rating for a seat cushion and the pressure distribution on the interface between the cushion and the buttocks. For the analysis of the contact pressure distribution, a finite element (FE) model of the human buttock was developed. The model consists of a detailed geometric description of the skin, soft tissues, and bony structures. The development of the model is described in this paper, which includes source data selection, bony structure modeling, joint modeling, soft tissue modeling, and pelvis shape morphing.

## INTRODUCTION

With combat bomber crew missions during Operation Enduring Freedom reaching over forty hours in length, the crewmember sitting comfort has become increasingly important to many of today's high-performance USAF fighter and tactical aircraft. Comfort is critical to both physical endurance and combat effectiveness.

Ejection seat cushions in current U.S. Air Force aircraft are not adequate for comfort during extended missions [1, 2]. Specific physiological problems resulting from or related to the discomfort of seating involve pain in the buttocks, legs and back; numbness and tingling in the extremities; and overall fatigue. Whereas a sophisticated circulation-enhancing seating system could provide substantial improvements in occupant comfort, it has limited application to military aircraft seats, especially ejection seats, as they are an integral part of an aircraft life support system. The introduction of any complicated systems or additional parts to enhance comfort would require extensive integration and qualification efforts at considerable cost. Therefore, solutions for comfort that can be quickly and cost-effectively implemented are desired. Fortunately, long-term sitting comfort can be enhanced by a new or improved seat cushion. A number of cushion designs with new materials and configurations have been introduced recently for the improvement of comfort.

Comfort is a subjective feeling influenced by psychological, physiological, and physical factors. However, experimental investigations have found that there exists a strong relationship between the human subjective discomfort rating for a seat cushion and certain

physical quantities of the pressure distribution on the interface between the cushion and the buttocks. These quantities include contact area, peak pressure, and the distribution center. The pressure distribution depends upon the cushion material and configuration. Thus, the comfort performance of a cushion can be improved by optimizing its material properties and configuration. Computational modeling and simulation of various designs can be an effective and efficient way to optimize the comfort performance of a cushion. A new design can be tested for its degree of comfort by computational simulations, which would reduce the amount of prototypes needed to introduce a new seat design. For the analysis of the contact pressure distribution, a finite element model of the human buttocks is required. The development of the model is described in this paper, which includes source data selection, bony structure modeling, joint modeling, soft tissue modeling, pelvis shape morphing, and model validation.

## EXPERIMENTAL INVESTIGATIONS OF SEATING COMFORT

To define the requirements for the FE human buttock model development, it is necessary and beneficial to have a review of recent seating comfort experimental investigations and the findings from them.

A series of cushion comfort tests were conducted at the Air Force Research Laboratory (AFRL) as a part of an overall effort to define seat cushion parameters that will maximize the comfort performance of a cushion without jeopardizing its safety performance. A pilot study was done in 1999 in which 5 males were monitored for a 4-hour sitting duration [3]. This study indicated the need for long-duration monitoring to gain a realistic understanding of the long-term effects on the operator's responses. The pilot study also led to improvements for the first 8-hour sitting duration study conducted in 2003 in which a larger, more diverse subject panel was observed on 4 cushion types in an F-15 seat configuration [4]. The 2003 study revealed that

correlations exist in objective seated pressures and subjective comfort levels. Based upon the previous studies, an expanded study was conducted in 2005 by introducing additional variables into tests, which included conventional cushions with static properties as well as new cushion designs with dynamic properties, increased measurement frequencies, and new measurement techniques [5]. These techniques included monitoring the change in lower extremity blood oxygen saturation levels to provide an estimation of blood flow behavior and monitoring low back and shoulder muscular fatigue. Blood pooling was selected for monitoring because periods of minimal to no motion in the leg in long-term flight could lead to deep vein thrombosis. Muscular fatigue levels in the low back and shoulder were selected to be monitored due to the long-duration effects of low-level sustained contractions. Combined, these factors were considered to be potentially significant contributors of discomfort during seated long-term flight.

The major findings and results from these tests can be summarized as follows.

- *Discomfort rating:* As in the cushion evaluation conducted by Stubbs et al. [4], it was expected that cushions with the lowest peak pressure points would show positive characteristics in subjective and objective tests and that cushions with the highest peak pressure points would show negative characteristics in the tests. For the static cushions, this proved to be the case for the correlation between average peak pressure and subjective discomfort survey ratings for the buttocks and thighs [5].
- *Task performance:* The results of task performance suggest that static cushion comfort does not have a negative impact on subject performance [4]; however, low dynamic cushion comfort may have a negative effect on the performance [5]. This leads to the conclusion that seat cushion comfort can be objectively measured, but its impact on the subject task performance is not very high.

- *Muscle fatigue:* For static cushions, trapezius muscle fatigue was exhibited throughout the 8-hour session for both male and female subjects but with varying time durations. The dynamic cushion elicited a unique response for both males and females due to the fact that no fatigue, and potentially recovery, occurred at every 2-hour interval. No measurable fatigue activity was present for the lumbar muscles for both static and dynamic cushions. This may be due to the lack of constraints placed on the assumed posture of the test subjects. More realistic aircraft scenarios with appropriate mobility restraints need to be investigated in future studies.
- *Oxygen saturation:* Although minimal changes of oxygen saturation and no differences between cushions were found for female subjects, males exhibited significantly decreased levels of oxygen saturation for all cushions. Motion and maintaining proper blood flow are necessary to mitigate long-term effects, such as the discomfort that the male subjects felt after standing. Monitoring oxygen saturation in the lower extremities is a relatively new modality for determining blood flow and pooling patterns. Oximeter data collection and processing techniques need further investigations.
- *Gender difference:* The differences in comfort preference and other objective measurements between genders were significant. Certain anthropometric factors such as body weight distribution may also cause the differences among test subjects.
- *Other factors:* Stress level, concentration level, and the micro-environment may have important effects on the comfort testing results, especially task performance scores.

## REQUIREMENTS AND CONSIDERATIONS

In summary, the interface pressure distribution between the seat and human body is related to the seating discomfort. It can be readily measured from tests. It can also be obtained

from computational simulation if the seat structure and the human subject are well modeled. While muscular fatigue and blood oxygen saturation are related to the seating discomfort, more investigations are needed to obtain consistent and definite relationships. They can be objectively measured in tests but cannot be readily determined from computational simulations, because in terms of the state-of-the-art of human modeling, it is very challenging to accurately model the stress/strain in muscles and blood flow in large regions of diverse tissues. The seating comfort varies with gender and certain human anthropometric factors related to seating contact area and sitting posture.

Several human models were developed in recent years for the analysis of seating comfort. Among them, an FE buttock model was developed using MADYMO [6]. The model includes a detailed anatomical description of the bony structures, such as iliac wings, sacrum, coccyx, and femora. The soft tissues, muscles, fat, and ligaments are lumped together and the skin is modeled separately. The geometry of the model is based on a post mortem human subject that was a 78-year-old male. One problem with using the model in ejection seat cushion comfort analysis is that the anthropometry of the model does not represent the US Air Force aircrew population. Since seating comfort is strongly related to the buttock soft tissues, the variation of buttock tissues with age could lead to large discrepancy in comfort requirements between the young and the old. Another problem is that the limited validation of the model prevents readily using the model for practical applications. Another FE model reported in [7] was developed based on the MRI scan data of a young, healthy male subject, intending to investigate stress-strain condition in deep tissues. The boundary conditions of the model were constrained corresponding to the particular loading conditions of the test subject and certain assumptions were used in the modeling. The model needs more validation and improvement. Since the seating comfort depends upon not only seat cushion but also backrest support, a full finite element occupant

model was developed [8]. The model is representative of a 50<sup>th</sup> percentile male in the sitting position and includes anatomically precise features such as leg and pelvic bones, hip joint ligaments, full spine, deformable thighs, hips and trunk. Inner organs and other outer body segments are modeled with rigid bodies linked with nonlinear kinematic joints. While the simulations of the model have achieved sound agreement with the tests of a small number of human subjects, the model lacks the flexibility to account for anthropometric variations and gender differences.

Therefore, we define reasonable requirements for the FE buttock model as follows:

- To be able to simulate the interface normal force (contact pressure distribution), interface shear force, and the stress in certain regions of human lower body;
- To be representative of the Air Force aircrew population, allowing for gender differences and anthropometric variations;
- To be able to consider various seating postures;
- To be generic, independent, and computationally efficient, leaving spaces for enhancement and expansion.

## MODEL DEVELOPMENT

Seating comfort modeling includes the modeling of the seat structure (seat cushion) and the modeling of the human subject. The modeling of a seat structure using FE is straightforward. The major task of static cushion modeling is the determination of cushion material properties. The modeling of a dynamic cushion may be more involved as the mechanism of a dynamic cushion needs to be properly described. The modeling of the human subject is a complex task and is the focus of this study.

For seating comfort analysis, usually only the buttocks and the upper legs that are in contact with the seat are modeled in detail. The rest of the body can be considered as rigid and can be described by a rigid multi-body model.

## Source Data Selection

One open source of human anatomical data is the Visible Human Dataset from the National Library of Medicine, National Institute of Health.

([http://www.nlm.nih.gov/research/visible/visible\\_human.html](http://www.nlm.nih.gov/research/visible/visible_human.html)). While this dataset provides a complete visual insight of the entire human body, it is totally unstructured and static. Large efforts are needed to create an FE human model from the dataset. With the time limit and condition restraints, we chose to use other data resources available to us to build the model to meet basic requirements for the seating comfort analysis. These include:

- *VAKHUM (Virtual Animation of the Kinematics of the Human) Database*: ([http://www.ulb.ac.be/project/vakhum/public\\_dataset/public-data.htm](http://www.ulb.ac.be/project/vakhum/public_dataset/public-data.htm)) Sponsored by European Commission, it is an open source for non-commercial use. The database has the data for bony structures only. The data of soft tissues and skin are not available.
- *CAESAR (Civilian American and European Surface Anthropometry Resource) Database*: It contains anthropometric variability of men and women, ages 18-65. Using three-dimensional (3D) laser scanning technology, human body anthropometric surface data were collected for each person in a standing pose, full-coverage pose and relaxed seating pose.
- *In-house Human Body Scan Data*: The data was collected at the AFRL on human subjects using a 3D laser scanner. If necessary, the data can be readily collected on a particular subject.

## Model Geometric Construction

The bony structure is primarily drawn from the VAKHUM database and approximately represents a 50<sup>th</sup> percentile male. As shown in shown in Fig.1, it includes the pelvis (sacrum, coccyx, ilium, pubis, and ischium) and femurs. The bony structure is modeled with solid elements and is assumed to be rigid, as the



deformation of soft tissues is the primary factor in the comfort problem. Since the sacrum and coccyx directly meshed from the volume data from VAKHUM were more complex than needed, a simplified sacrum-coccyx component was created and meshed with solid elements, as shown in Fig. 2.

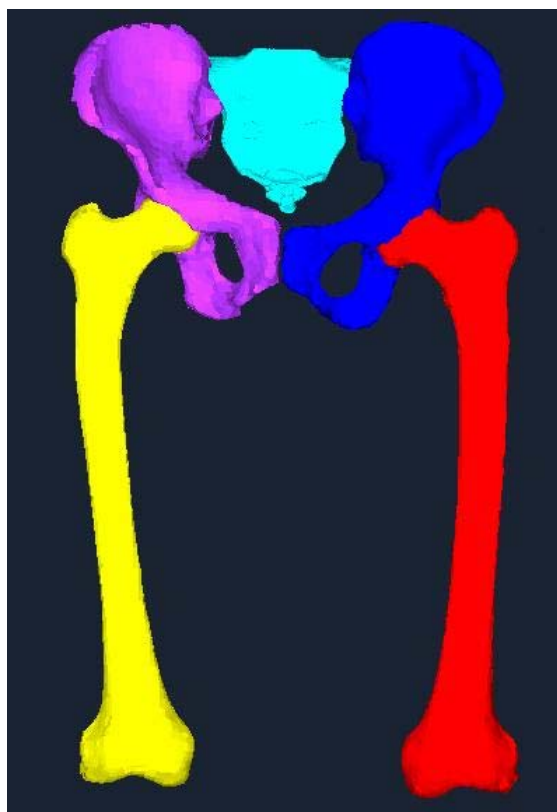


Figure 1. The bony structure of pelvis and femurs from the VAKHUM

Spherical joints, a joint type provided in LS-DYNA, have been applied between the iliac wings and the upper body, and between each iliac wing and femora (hip joints). The implementation of hip joints is to allow for the investigation of various sitting postures. By rotating the femur bones about the hip joints, the bony structure in a sitting posture is generated, as shown in Fig. 3.

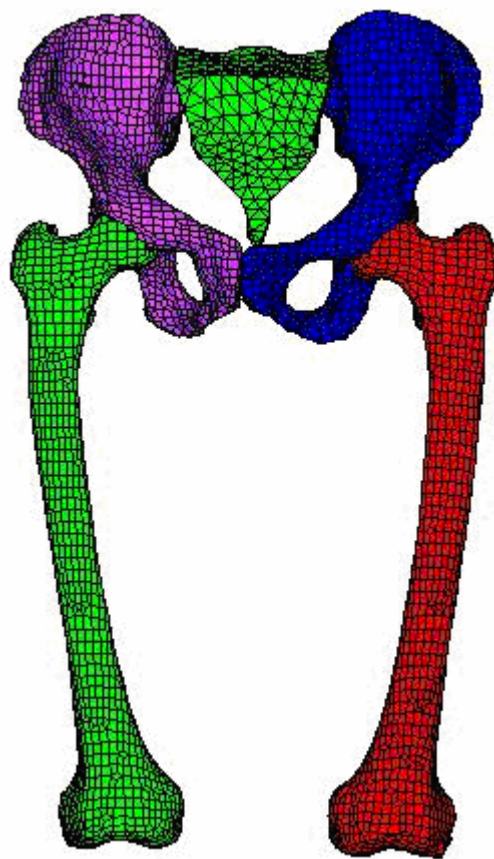


Figure 2. Solid-meshed bony structure with simplified sacrum-coccyx

Based on an in-house dataset, the outer skin shape was taken from a 3D anthropometric scan of a human male subject, approximately 50<sup>th</sup> percentile, in a standing position as shown in Fig. 4. The scan was cut just above the waist and just below the lateral epicondyles at the knees. The landmarks used for the bispinous breadth and bitrochanteric breadth anthropometric measurements projected slightly above the skin surface, and so were visible in the scan data. These measurements were used in scaling and positioning the bones and outer surface.

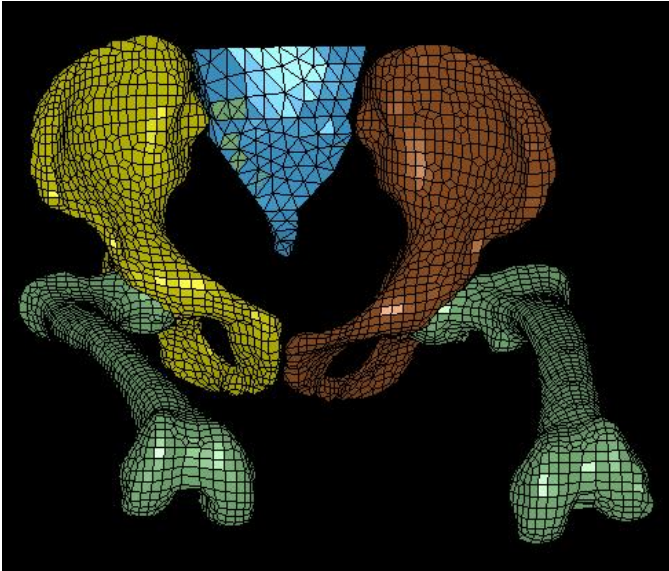


Figure 3. The bony structure in a sitting posture

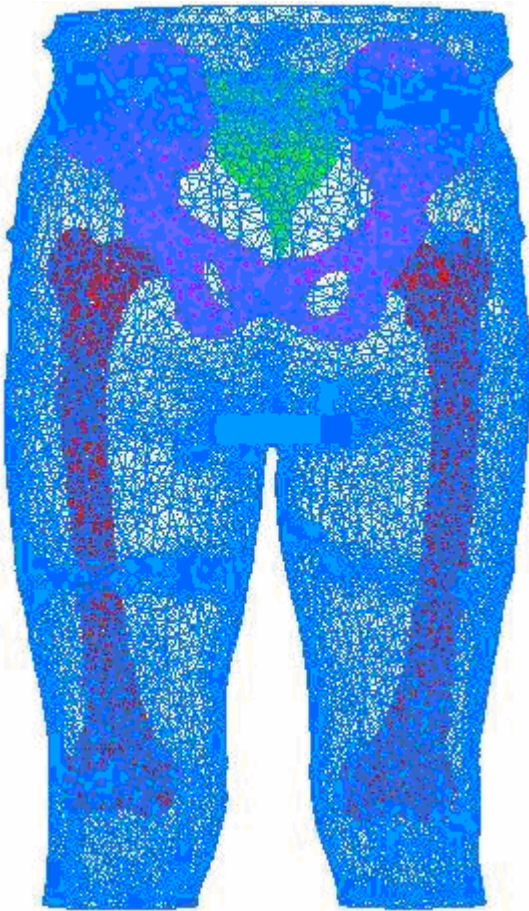


Figure 4. Three-dimensional anthropometric scan of a male subject (standing position)

For the analysis of seating comfort, the model needs to describe the human subject in a seated position. It is desirable to use a 3D scan of a seated human to generate the seated shape. However, the 3D scan surface of the same subject in a sitting position is not complete, as shown in Fig 5, because body surfaces hidden by the seat or by other body parts are not included. However, using the seated scan dimensions can approximate the changes in body shape from the standing to the seated position. The standing scan surface was modified to approximate the seated scan, including rotation of the legs.

Instead of modeling the buttock soft tissues according to detail anatomical data, the fat, muscles, and ligaments are lumped together and are described by layers of solid elements. The soft tissue modeling started with the thigh, due to the relatively simple geometry. A cone, truncated at both ends and sized to a slightly smaller diameter than the outer "skin", was created from lines, then meshed with shell elements using HyperMesh from Altair Engineering. The volume of the cone was meshed with layers of solid elements to fill the space as much as possible down to the femur. Then the outer shell layer of the soft tissue cone was morphed to the 3D scan surface. It was necessary to create regular-shaped layers of solid elements before morphing because the process of adding layers does not work well after morphing. The inside of the soft tissue was morphed to the surface of the femur shaft, making certain nodes of soft tissue elements coincide with the nodes of femur solid elements. This worked well at the distal end, but left a gap at the proximal end because of its larger diameter. Manual adjusting was used to improve the soft tissue fill at the proximal end.

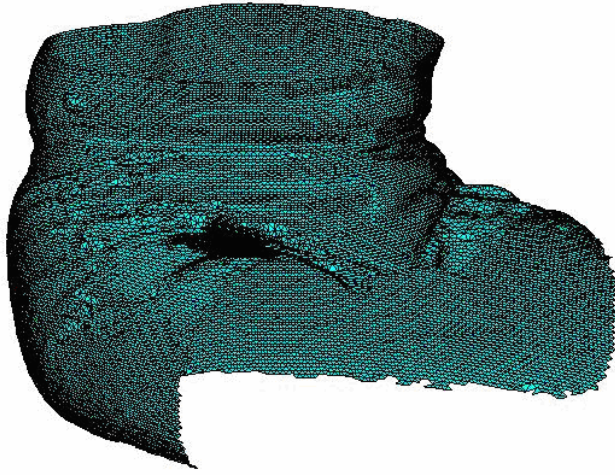


Figure 5. Three-dimensional anthropometric scan of a male subject (seated position)

For the pelvic volume, the soft tissue creation was more complex. Due to the irregular shape of the pelvis bony structure, the space between the outer skin and the bones is more complicated than the thigh. Many commercial FE meshing tools are unable to generate or recognize the volume of the spaces of this kind and thus automatic mesh generation becomes impossible.

Two methods can be used to overcome this difficulty. One is to create the volume by a Boolean subtraction of the bone volumes from the 3D scan surface volume in the buttock region. This was accomplished using Pro/ENGINEER. The entire volume was further segmented into several sub-volumes with simple shapes and then imported to HyperMesh and solid meshed.

Another method is to divide the entire space into two regions—anterior side and posterior side, and to treat them differently. The viscera filling the anterior side of the pelvis contains organs with fluids. It is more easily deformed and moved. Therefore, it can be modeled with shell-element bags of viscous fluid, or with SPH elements in LS-DYNA. The posterior buttock region, which plays a more important role than the anterior region in the seating comfort, needs to be modeled in detail with

solid elements. This was done by segmenting the region into several sub-regions and then meshing each sub-region manually.

### **Material Properties**

The material properties of the model are initially taken from values in the open literature. The bony structure is assumed to be rigid. Thus Material Type 20 in LS-DYNA is chosen with the parameters of Young's modulus  $E = 10$  GPa, Poisson's ratio  $\mu = 0.3$ , and mass density  $\rho = (1.1 \sim 1.2) \times 10^3$  kg/m<sup>3</sup>, which varies slightly with each bone part. The skin is described by a linear elastic isotropic material model (Material Type 1 in LS-DYNA), with the parameters of  $E = 0.85$  MPa,  $\mu = 0.46$ , and  $\rho = 1.1 \times 10^3$  kg/m<sup>3</sup> [6,9]. For the soft tissues, Mooney-Rivlin hyperelastic isotropic material model (Material Type 27 in LS-DYNA) is used. According to the description of this material model [10], the parameters are chosen as  $A_1 = 1.65$  kPa,  $B = 3.35$  kPa, and  $\mu = 0.49$ , and thus  $C = 4.175$  kPa, and  $D = 51.225$  kPa. For each layer of soft tissues, these values are allowed to vary in a small range.

### **Model validation**

The model is still under construction. The completed model will be validated by comparing simulation results with test data.

## **CONCLUDING REMARKS**

The comfort performance of a cushion can be improved by optimizing its material properties and configuration. Computational modeling and simulation of various designs can be an effective and efficient way to optimize the comfort performance of a cushion.

Whereas an FE human buttock model was developed in this paper, more work on the model is to be done in order to use it for practical applications, which includes the model validation, modification, and refinement. To scale the base model, especially the buttock outer shape to represent a particular



test subject according to his or her 3D laser scan data, is one of our interests and will be investigated in the future.

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